MATH0018 Functional Analysis

Year:	2024{2025
Code:	MATH0018
Level:	6 (UG)
Normal student group(s):	UG: Year 3 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0051
Lecturer:	Dr M Karpukhin

Course Description and Objectives

Elementary analysis mostly studies real-valued functions on the real line R or on n-dimensional space R^n. Functional analysis, by contrast, shifts the point of view: we collect all the functions of a given class (for instance, all bounded continuous functions) into a *space of functions*, and we study that space (and operations on it) as an object in its own right. Since spaces of functions are nearly always in nite-dimensional, we are led to study analysis on in nite-dimensional vector spaces, of which the most important cases are Banach spaces and Hilbert spaces. This course provides an introduction to the basic concepts of functional analysis. These concepts

- Normed linear spaces and Banach spaces. Examples: Sequence spaces ℓ^p $(1 \le p \le \infty)$ and c_0; spaces $C(X)$ of bounded continuous functions. Proofs of completeness of these spaces. Special properties of finite-dimensional normed linear spaces.
- Hilbert spaces. Basic properties. Orthogonal systems and the orthogonalization process. Representation of linear functionals on Hilbert space.
- Zorn's lemma and the Hahn-Banach theorem.
- Linear functionals and duality. Dual of ℓ^p is ℓ^q. Second dual and reflexive spaces.
- Baire category theorem. Uniform boundedness theorem, open mapping theorem, closed graph theorem.
- Weak and weak-$*$ topologies. Weak-$*$ compactness of the unit ball in the dual space. Compact operators.